Counting

Emergent

During the Emergent phase:

…students come to understand that number words and symbols can be used to signify the "numerosity" of a collection

By the end of the Emergent phase:

- Use "bigger", "smaller" and "the same" to describe differences between small collections
- See at a glance how many are in small collections

These students recognize that numbers may be used to signify quantity.

in the MATCHING Phase

Matching

Most students enter this phase between 3 and 5 years of age

As students enter the Matching phase:

- May say a string of the number names in order, but not connect them with how many are in the collection
- Can tell by looking which of two small collections is bigger; (but not) how much bigger

During the Matching phase:

...students use numbers as adjectives to describe actual quantities of physical material

By the end of the Matching phase:

- Know how to count a collection, respecting most of the principles of counting
- Compare two collections one to one and use this to decide which is bigger and how much bigger
- Solve small number story problems

These students use one-to-one relations to share and count out.

in the QUANTIFYING Phase

Quantifying

Most students enter this phase between 5 and 6+ years of age

As students enter the Quantifying phase:

- Often still think they could get a different answer if they started at a different place, so do not trust counting on or counting back
- Often do not spontaneously use counting to compare two groups
- Often can only solve addition and subtraction problems which they can directly represent or imagine

During the Quantifying phase:

…students see that the significance of the number uttered at the end of the counting process does not change with rearrangement of the collection or the counting strategy

By the end of the Quantifying phase:

- Find it obvious that counting on will give the same answer as starting at the beginning
- Can think of addition and subtraction situations in terms of the whole and the two parts and which is missing
- Understand that the more portions to be made from a quantity, the smaller the size of each portion

These students use part-part-whole relations for quantities.

in the PARTITIONING Phase

Partitioning

Most students enter this phase between 6 and 9 years of age

As students enter the Partitioning phase:

- Often cannot decompose numbers that they cannot visualize or represent as quantities
- May be unable to use the inverse relationship between addition and subtraction to choose the more efficient (strategy)
- Often do not link sharing to unit fractions and may think that eighths are bigger than thirds because 8 is bigger than 3

During the Partitioning phase:

…students see that numbers have magnitudes in relation to each other, can interpret any whole number as composed of two or more other numbers, and see the relationship between different types of addition and subtraction situations

By the end of the Partitioning phase:

- Partition numbers into standard component parts (326 = 300 + 20 + 6) without reference to actual quantities
- Use the inverse relationship between addition and subtraction to calculate
- Can double count in multiplicative situations simultaneously keeping track of the number of groups and the number in each group

These students use additive thinking to deal with many-to-one relations.

in the FACTORING Phase

Factoring

Most students enter this phase between 9 and 11 years of age

As students enter the Factoring phase:

- Often do not realize that the digit in the tens (hundreds) place refers to groups of tens (hundred) even when they correctly use the labels "ones", "tens", "hundreds"
- Often are unable to use the inverse relationship between division and multiplication to work out an unknown quantity
- May think of fractions as quantities rather than numbers

During the Factoring phase:

...students extend their additive ideas about whole numbers to include the coordination of two factors needed for multiplicative thinking

By the end of the Factoring phase:

- Find it obvious that if 3 rows of 5 is 15, then both 15 divided by 3 and one-third of 15 are 5
- Know that they can choose between multiplication and division to make calculating easier
- Relate fractions and division, knowing 3 things shared among 4 students has to be 3/4

These students think both additively and multiplicatively about numerical quantities.

in the OPERATING Phase

Operating

Most students enter this phase between 11 and 13 years of age

As students enter the Operating phase:

- Often believe that multiplication "makes bigger" and division "makes smaller"
- May resist selecting division where the required division involves dividing a number by a bigger number
- Often ignore the need to draw two fractions on identical wholes in order to compare and combine them

During the Operating phase:

…students learn to make multiplicative comparisons between numbers, deal with proportional situations, and integrate their ideas about common and decimal fractions

By the end of the Operating phase:

- Are flexible in partitioning decimal numbers
- Realize that for multipliers smaller than 1, multiplication makes smaller and for divisors smaller than 1, division makes bigger
- Can write suitable (mathematical models) for a full range of 'x' and '÷' situations involving whole, fractional and decimal numbers

These students can think of multiplications and divisions in terms of operators.

At the end of the OPERATING Phase

Additive Thinking

Multiplicative Thinking

PEARSON Professional Learning

first steps ®

First Steps in Mathematics

Overview of Diagnostic Map: Number

S06-047-BC