CASE STUDY 1

Sample Learning Activity: K–Grade 3—Snail Trails, pages 56, 72
Grades 3–5—Snail Trails, page 44

Key Understanding 3: To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.

Working Towards: Matching and Comparing Phase and the Quantifying Phase

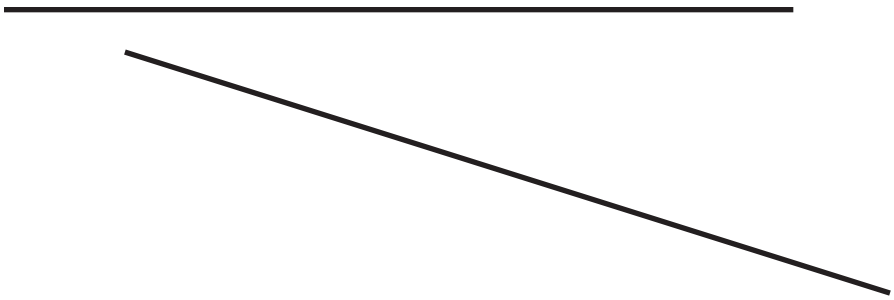
TEACHERS' PURPOSE

Students in my Grade 3 class had begun to use units to measure length and seemed to be able to do it. For example, Roberto chose straws to measure the length of the bookcase and said, "The bookcase is a bit more than 12 straws long."

I was not sure, however, whether they would use this information to compare the length of two different objects and decided that this should be the focus of my next measurement lesson.

MOTIVATION AND PURPOSE

I provided sheets of paper on which I had drawn two straight lines to represent snail trails across a path. Each page was a little different, so students had to work with their own pair of trails.



I gave each pair several different types of materials to use as units from among toothpicks, paper clips, marbles, blocks and beans, but made sure that there was only enough of any one material to measure one of the trails, not both. I had a reason for doing this: I wondered if they really knew that the count was an indication of the length of the object. I did not give them enough objects of the same size to match both lines at the same time because I wanted to create conflict that would provoke the students to think about what to do. My goal was that they would work out that they could use the objects to measure one line, remember the number, and then place the same objects onto the other line.

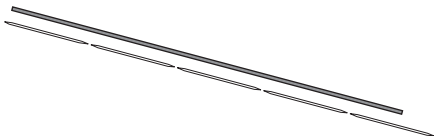
I then asked, "Which snail went farther and how much farther did this snail go than its friend?"

ACTION AND REFLECTION

Almost all students began by choosing one of the units and repeating it by lining up the units end-to-end along the first trail. Many of them, however, happily chose a different-sized object (and hence unit) to measure the second trail. I saw Adam run out of toothpicks and begin using blocks that were a different size halfway along the second line. "There wasn't enough, so I had to use another one." After seeing what others in his group had done, he replaced the toothpicks on the second line with blocks.

Even though I had anticipated that some students would choose to use different units, I was surprised that so few students thought about the need for the same unit and so did not feel any conflict about using different units. Having counted the units, most disregarded the object they had used and simply compared numbers.

I encouraged students to reflect on the sense of what they had found. For example, Kim used paper clips for the horizontal line and toothpicks (which were a different size to the paper clips) for the other.



I asked him why he chose the toothpicks. He said, "They are skinny and they fit on the line better and I can see if they are straight on it."

I then asked him which of his trails was longer and he immediately pointed to the diagonal line. Then, I asked, "How much longer?"

Kim counted the units on each line, frowned, looked a bit confused, counted again and said, "That's six, there's only five there, so it's one more?"

I asked why he had frowned and counted again. He said, "I thought I was wrong because the five one's longer, but I counted again and it is one more there."

I helped Kim notice an anomalous or conflicting result without actually correcting him or simply telling him "how to do it."

CONNECTION AND CHALLENGE

I realized that Kim did not understand how counting the units related to the length. He carefully lined the units up end-to-end as I had taught him, but did not seem aware that the differing lengths of the two units would interfere with the comparison. He also ignored the "part-unit" left on one line. His hesitation, however, suggested he might be ready to move on, so I suggested he try swapping the toothpicks and paper clips.

Kim's confusion about what he has found provides an opportunity for him to make a significant leap in his knowledge.

Kim did this and found that there were seven paper clips and barely five toothpicks. "Ah," he said, "It is two more, that has to be right—the long one's got more this time."

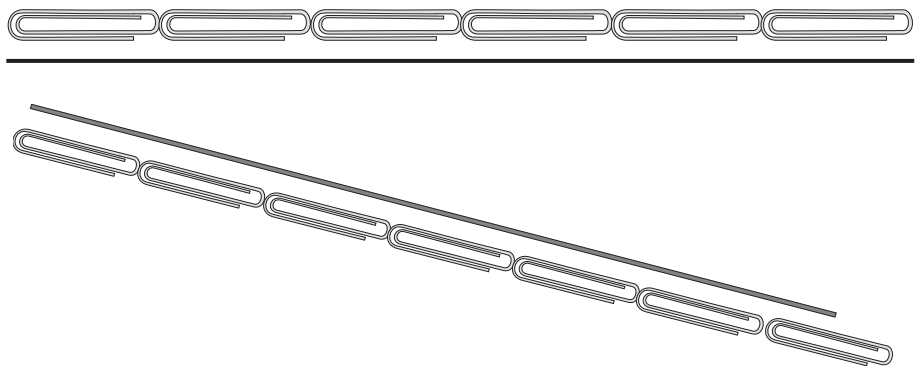
He was clearly more satisfied with this result because the greater number agreed with his knowledge about which line was longer.

But I wanted him to go further, and focused back on the type of units. "So there are 7 paper clips on the longer line, but how many toothpicks did you say fitted on the short line when you did it the other way?"

At this point, the nature of the teacher's input is crucial.

At this point, Kim paused, studying his trails intently, while I waited. At last, he answered, saying, "It was six." But then he added in an excited voice, "You do not need toothpicks, you just need paper clips and then you do not get messed up. It is paper clips on this line and paper clips on that line and then it makes sense. It is 7 paper clips on the long line and this one was 6 paper clips, so the long line is more, it fits one more paper clip."

Catherine had helped Kim begin to make a connection between the length unit and the attribute it measured.



I wanted Kim to see that either the paper clips or the toothpicks could be used to make the comparison, but not both at the same time.

DRAWING OUT THE MATHEMATICAL IDEA

I gave Kim a different coloured felt pen and had him write down what he had found on his sheet of paper

I then paused and said slowly, "So you are saying that this one is 6 paper clips long but this one is 7 paper clips (pointing) and it is the longer one. What about when you measure both with the toothpicks?" I left that with him.

CASE STUDY 2

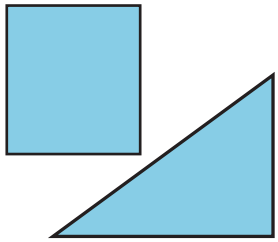
Sample Learning Activity: Grades 3–5—Garden Plots, page 59; Grades 5–8—Garden Plots, page 46, 140

Key Understanding 3: To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.

Working Towards: Quantifying Phase and Measuring Phase

MOTIVATION AND PURPOSE

I began by wanting to get my Grade 4 students thinking about what sorts of things were useful for representing a unit of area. (See Key Understanding 4, Grades 3–5 Sample Learning Activity "Garden Plots.") I asked them to compare two different-shaped garden plots and gave them a range of things to choose from to use to represent their unit, including toothpicks, 1-cm cubes, 2-cm cubes, pattern blocks, rice, counters, and string.



Which garden plot has the most land?Choose the plot with the larger area so we can grow more vegetables on it.

The rectangle was exactly 11 cm by 12 cm so that 2-cm cubes would not fit exactly into it. The right-angled triangle was 17 cm by 15 cm. I chose regions that differed in area by just a few square centimetres so that the students could not say which was larger simply by looking and would have to find another way to compare them.

CONNECTION AND CHALLENGE

When the students began work, I circulated, observing their strategies and asking questions. I noticed that a number of the students made their choice of measuring materials according to the shape of the region they were trying to measure and used different materials on one region than on the other. The idea of "fitting" dominated their thinking. Joshua said, "I have used blocks here, because it is a square and rice here because the blocks do not fit and rice fits better." When I asked him if he had found out which was larger, he just stared at his carefully placed blocks and rice for a few minutes, then looked at me rather helplessly and shook his head.

Other students, like Tilopa, used blocks on both regions, but filled in the gaps around the edges with rice. Asked which was bigger, she counted the blocks but had difficulty when she started to count grains of rice. Thus, some students did not see that using the same material on both regions would help them to make the comparison. Others saw this and started out using the same material for both regions but had difficulty with the gaps around the edges. I decided to focus on using the same material first, rather than the gaps.

ACTION AND REFLECTION

I stopped the students and asked, "So, which region is bigger?"

Most agreed with Joshua, who said you could not tell. "Why not?" I asked.

"Well," said Joshua, "you have to put different things on, blocks for the square and rice for the triangle."

"Why did you have to use different things, Joshua?"

"Because they are different shapes, so some stuff will not fit on," he replied.

"So what can we do to find out which is larger?"

I noticed earlier that Brendon used the same material on both but had difficulty covering the region with the materials provided. "Brendon, did you find out which one was larger?"

"Well, sort of," he said, "I think the square might be bigger."

"What makes you think that?" I asked.

"When I put blocks on both of them I had 30 blocks on the square and only 29 on the triangle. But the problem is that there was lots left over and so I could not tell really."

"You managed to use blocks on both of them." I said this because I wanted the students to begin to think about using the same material for both regions. "Did anyone else cover both regions with the same thing?"

Halimah volunteered that she had used rice on both but could not tell which was bigger because it was too hard to count.

"But if you did count it, would you be able to tell which was the biggest?" I asked.

"Yes," she said.

"So, how would you know which was biggest?"

"Because the one with the biggest number would be the biggest one."

"So, why can't you tell which is biggest when you count what you have, Joshua?" I wanted the students to think about the importance of using the same unit on both regions.
"Because it is different when you have different stuff," he said.
"I know," said Tilopa. "It's because they are not the same size. You see, rice is much smaller than blocks and so 30 pieces of rice is much smaller than 30 blocks."

DRAWING OUT THE MATHEMATICAL IDEA

"Yes, that is important," I said. "You have to use the same thing on both of the shapes, otherwise you can't use the number to tell you which is bigger."

I asked them to work with their partner to say what we needed to do, and then asked for a volunteer to say it to the class. I then repeated this for the whole class: "Choose one type of material to use as your unit and cover both shapes with it. Count how many cover each shape. Then the numbers will tell which is bigger."

They started the task again, all using the one material of their choice. I was not sure that all really understood the importance of using the same unit in order to compare, but they were on the way. I knew that many would experience the problem that Brendon and Tilopa had found, with fitting their units along the edges. This I would deal with later.