# Diagnostic Map: Measurement

## Emergent Phase

### During the Emergent Phase

Students initially attend to overall appearance of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small) rather than to describe comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgements of size.

**As a result, they begin to understand and use the everyday language of attributes and comparison used within their home and school environment, differentiating between attributes that are obviously perceptually different.**

### By the end of the Emergent phase, students typically:

- distinguish tallness, heaviness, fatness, and how much things hold
- start to distinguish different forms of length and to use common contextual length distinctions; e.g., distinguish wide from tall
- use different bipolar pairs to describe things; e.g., thin—fat, heavy—light, tall—short
- describe two or three obvious measurement attributes of the same thing; e.g., tall, thin, and heavy
- describe something as having more or less of an attribute than something else, e.g., as being taller than or as being fatter than.

## Matching and Comparing Phase

Most students will enter the Matching and Comparing phase between 5 and 7 years of age.

### As students move from the Emergent phase, to the Matching and Comparing phase, they:

- may not "conserve" measures; e.g., thinking that moving a rod changes its length, pouring changes "how much," cutting up paper makes more surface area
- may visually compare the size of two things, but make no effort to match; e.g., saying which stick is longer without lining up the bases or which sheet of paper is bigger without superimposing
- compare time spans but may not take into account different starting times; e.g., deciding that the TV program that finished latest was on longest
- use bipolar pairs but may have difficulty with some comparative terms; e.g., lift to decide which is heavier but say both are heavy because both hands go down
- may distinguish two attributes (such as tallness and mass) but not understand that the two attributes may lead to different orders of size for a collection, expecting the order for tallness and the order for mass to be the same
- while describing different attributes of the same thing (tall, thin, and heavy) may be confused by a request to compare two things by different attributes, particularly if the comparisons lead to different orders
- often do not think to use counting to say how big or how much bigger; e.g., they may "weigh" something by putting it into one side of a balance and smaller objects into the other side but not count the objects

### During the Matching and Comparing Phase

Students match in a conscious way in order to decide which is bigger by familiar readily perceived and distinguished attributes such as length, mass, capacity, and time. They also repeat copies of objects, amounts, and actions to decide how many fit (balance or match) a provided object or event.

**As a result, they learn to directly compare things to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as "How long (tall, wide or heavy, much time, much does it hold)?"or when explicitly asked to measure something.**

### By the end of the Matching and Comparing phase, students typically:

- attempt to focus on a particular attribute to compare two objects or events; e.g., how much the jar holds
- know that several objects or events may be in different orders when compared by different attributes
- line up the base of two sticks when comparing their lengths and fit regions on top of each other to compare area
- use the everyday notion of "how many fit" and count how many repeats of an object fit into or match another; e.g., How many pens fit along the table? How many potato prints cover the sheet? How many blocks fit in the box?
- count units and call it "measuring;" e.g., *I measured and found the jar holds a bit more than 7 scoops*.
- use "between" to describe measurements of uni-dimensional quantities (length, mass, capacity, time); e.g., *It weights between 7 and 8 marbles*.
- refer informally to part-units when measuring uni-dimensional quantities; e.g., *Our room is 6 and a bit metres long*.

## Quantifying Phase

Most students will enter the Quantifying phase between 7 and 9 years of age.

### As students move from the Matching and Comparing phase to the Quantifying phase, they:

- while knowing that ordering objects by different attributes may lead to different orders, may still be influenced by the more dominant perceptual features; e.g., they may still think the tallest container holds the most
- may count "units" in order to compare two things but be fairly casual in their repetition of units, not noticing gaps or overlaps; e.g., placing the first "unit" away from the end when measuring length, not worrying about spills when measuring how much a container holds, not stopping their claps immediately the music stops
- do not necessarily expect the same "answer" each time when deciding how many fit
- many not think to use unit information to answer questions such as: Which cup holds more? Will the table slide through the door?
- may not see the significance of using a common unit to compare two things and, when using different units, let the resulting number override their perceptual judgement
- while many will have learned to use the centimetre marks on a conventional rule to "measure" lengths, they often do not see the connection between the process and the repetition of units

### During the Quantifying Phase

Students connect the two ideas of directly comparing the size of things and of deciding "how many fit" and so come to an understanding that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared without directly matching them.

**As a result, they trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.**

### By the end of the Quantifying phase, students typically:

- attempt to ensure uniformity of representations of the unit; e.g., check that the cup is always full, the pencil does not change length, the balls are the same size
- use the representations of their unit carefully to make as close a match as possible, avoiding gaps and overlaps; e.g., choose a flexible tape to measure the perimeter of a curved shape
- know why they need to choose the same size objects to use as units when comparing two quantities
- see repeating one representation of the unit over and over as equivalent to filling or matching with multiple copies of it
- connect the repetition of a 'unit' with the numbers on a whole-number calibrated scale
- make things to a specified length in uniform units (including centimetres and metres)
- use provided measurements to make a decision about comparative size; e.g., use the fact that a friend's frog weighs 7 marbles to decide whether their own frog is heavier or lighter
- count units as a strategy to solve comparison problems such as: Whose frog is heavier? Put the jars in order from the one that holds the most to the one that holds the least.
- are prepared to say which is longer (heavier) based on information about the number of units matching each object
- think of different things having the same "size"; e.g., use grid paper to draw different shapes with the same perimeter
- add measurements that they can readily think of in terms of repetitions of units; e.g., find the perimeter of a shape by measuring the sides and adding

## What is the Diagnostic Map for Measurement?

How students currently think about measurement attributes and units will influence how they respond to the activities provided for them, and hence what they are able to learn from them. As students' thinking about measurement develops, it goes through a series of characteristic phases. Recognizing these common patterns of thinking should help you to interpret students' responses to activities, to understand why they seem to be able to do some things and not others, and also why some students may be having difficulty in achieving certain outcomes while others are not. It should also help you to provide the challenges students need to move their thinking forward, refine their half-formed ideas, overcome any misconceptions they might have to and hence achieve the outcomes.

See over

# Diagnostic Map: Measurement cont.

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## Measuring Phase

Most students will enter the Measuring phase between 9 and 11 years of age.

### As students move from the Quantifying phase to the Measuring phase, they:

- while trying to make as close a match as possible to the thing to be measured, may find the desire to match closely overriding the need for consistency of unit; e.g., they may resort to "filling" a region with a variety of different objects in order to cover it as closely as possible
- may not understand that the significance of having no gaps and overlaps is that the "true" measurement is independent of the placement of the units
- may still think of the unit as an object and of measuring as "fitting" in the social sense of the word (How many people fit in the elevator? How many beans in the jar?) and so have difficulty with the idea of combining part-units as is often needed in order to find the area of a region
- many confuse the unit (a quantity) with the instrument (or object) used to represent it; e.g., they may think a square metre has to be a square with sides of 1 m, may count cubes for area and not think of the face of each as the unit
- may interpret whole numbered marks on a calibrated scale as units but may not interpret the meaning of unlabelled graduations

### During the Measuring Phase

Students come to understand the unit as an amount (rather than an object or a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

**As a result, they see that part-units can be combined to form whole units and they understand and trust the measurement as a property or description of the object being measured that does not change as a result of the choice or placement of units.**

### By the end of the Measuring phase, students typically:

- expect the same number of copies of the representation of their unit to match the object being measured regardless of how they arrange or place the copies
- understand that the smaller the unit the greater the number; e.g., are able to say which is the longer of a 1-km walk and a 1400-m walk.
- compose "part-units" into wholes, understanding, for example, that a narrow garden bed may have an area of 5 or 6 $m^2$ even though no whole "metre squares" fit into the bed
- can themselves partition a rectangle into appropriate squares and use the array structure to work out how many squares are in the rectangle
- interpret the unnumbered graduations on a familiar whole-number scale
- understand the relationship between "part-units" and the common metric prefixes; e.g., know that a unit can be broken into one hundred parts and each part will be a centi-unit
- work with provided measurement information alone; e.g., order measurements of capacity provided in different standard units, make things that meet measurement specifications

## Relating Phase

Most students will enter the Relating phase between 11 and 13 years of age

### As students move from the Measuring phase to the Relating phase, they:

- while partitioning a rectangle into appropriate squares and using the array structure to find its area, may not connect this with multiplying the lengths of the sides of a rectangle to find its area
- while understanding the inverse relationship between the unit and the number of units needed, may still be distracted by the numbers in measurements and ignore the units; e.g., say that 350 g is more than 2 kg
- while converting between known standard units, may treat related metric measures just as they would any other unit, not seeing the significance of the decimal structure built into all metric measures

### During the Relating Phase

Students come to trust measurement information even when it is about things they cannot see or handle and to understand measurement relationships, both those between attributes and those between units.

**As a result, they work with measurement information itself and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.**

### By the end of the Relating phase, students typically:

- understand that known relationships between attributes can be used to find measurements that cannot be found directly; e.g., understand that we can use length measurements to work out area
- know that for figures of the same shape (that is, similar) the greater the length measures the greater the area measures, but this is not so if the figures are different shapes
- understand why the area of a rectangle and the volume of a rectangular prism can be found by multiplying its length dimensions and can use this for fractional side lengths
- think of the part-units themselves as units; e.g., a particular unit can be divided into one hundred parts and each part is then a centi-unit
- subdivide units to make measurements more accurate
- choose units that are sufficiently small (that is, accurate) to make the needed comparisons
- use their understanding of the multiplicative structure built into the metric system to move flexibly between related standard units; e.g., they interpret the 0.2 kg mark on a scale as 200 g
- notice and reject unrealistic estimates and measurements, including of objects or events they have not actually seen or experienced
- use relationships between measurements to find measures indirectly; e.g., knowing that 1 mL = 1 $cm^3$ they can find the volume of an irregular solid in cubic centimetres by finding how many millilitres of water it displaces using a capacity cylinder