Diagnostic Map: Geometry and Space

Emergent Phase

During the Emergent Phase

As students move about their environment and explore the objects in it, they respond perceptually to spatial features, encoding shape and the location of objects they can see within a framework of landmarks.

As a result, they begin to name things they can see and touch in ways that reflect attention to shape and they can match simple shapes in an impressionistic way.

Also as a result, they begin to understand that we can represent the relative position of neighbouring things, for example, placing a toy boy under the toy table to "stand for" the real boy under the real table.

By the end of the Emergent phase, students typically:

- distinguish shape from other attributes that relate to how things "look" (e.g., colour, size, texture), although they may not do so consciously
- use informal language that indicates they are responding to shape (e.g., "the pointy one")
- carry out matching tasks by selecting a matching shape from a collection and either posting shapes in boxes or fitting shapes into cut-outs
- notice similarity in the shape of familiar things, saying, for example, it "looks like" a see-saw or a car or a ball
- reproduce simple geometric configurations if only encoding is required; that is, build a matching shape or arrangement to one that is constantly in sight
- draw simple figures by imitating how they have seen them drawn (including letters and numbers)
- give directions from one landmark to the next when retelling a journey or places in a story (e.g., "go to the pond, go on the bridge, go home")

Recognizing Phase

Most students will enter the Recognizing phase between 4 and 5 years of age.

As students move from the Emergent phase, to the Recognizing phase, they:

- refer to objects by their everyday or toy names (blocks, bricks, party hat) rather than their shape
- may not think to turn a figure over or around in order to match or post in cut-outs
- may have difficulty in matching a shape by feel alone (e.g., in feely boxes) as they group and pat objects rather than explore in a way oriented to discerning shape
- given drawings, will not distinguish, for example, triangles from "almost triangles," relying on an impressionistic match
- may be able to copy a figure such as a square with toothpicks but not be able generally to copy one from a ready made drawing unless shown how (that is, they have difficulty in dissecting the parts and deciding the sequence or route to bring the components together)

These are the learning challenges for the Recognizing phase.

During the Recognizing Phase

Students' exploration of objects and space through touch and sight gradually becomes more regulated as they attend to spatial features and construct mental and visual representations of shapes and arrangements in space.

As a result, they can copy simple figures and recognize figures of "the same shape," constructing visual images or prototypes of what people mean when they refer to common figures and objects (e.g., *This is a rectangle because it looks like a rectangle.)*

Also as a result, they construct visual images of familiar objects and of where objects are within familiar spaces and locations.

By the end of the Recognizing phase, students typically:

- describe figures and objects using terms that are evocative of shape, such as "corner," "pointy," "lopsided," "slanty"
- learn the names of some shapes (triangle, cube) although which names they know will depend upon the frequency and naturalness of everyday use at school and home
- describe conventional figures and objects by reference to prototypes they "look like"; e.g., "It's a door shape."
- select ready-made materials that "look right" to make recognizable models of parts of their environment (e.g., circular pieces for wheels, a cylinder for a tree trunk)
- remember what some families of shapes look like and produce recognizable versions; e.g., draw a figure that resembles a triangle with three lines that more or less join at their end points or as a continuous curve with three "straight" sides and three corners
- remember key aspects of the way things look and try to reproduce them in their drawings; e.g., drawing circles for wheels; putting two eyes, a nose, and a mouth on a face
- begin to give simple explanations that relate shape to purpose (e.g., circles for wheels, blocks to stack)
- relate the position of objects to each other in familiar settings using terms such as "behind," "near"
- draw or make simple "route" maps and models that show a sense of spatial relationships and order, although only for local settings that they have freely explored

Describing Phase

Most students will enter the Describing phase between 6 and 7 years of age.

As students move from the Recognizing phase to the Describing phase, they:

- explore objects using touch (e.g., in a feely box) oriented to shape as a whole, not generally focused
- while implicitly knowing some features of a familiar type of figure (triangles have three sides, triangles are pointy), do not recognize them in that way; a triangle is a triangle because it looks like one
- can identify familiar shapes singly but not within complex configurations or in non-standard orientations
- use terms such as "corner," "pointy," "lopsided," "slanty" vaguely and inconsistently
- are not consciously aware of properties; e.g., they could produce a recognizable rectangle without realizing that it had right angles
- although able to perceive the difference between a 2-D and 3-D thing, may think the word "shape" refers to a 2-D attribute and so may say there is no difference in the shape of a ball and a hoop, since both are called by the same shape name "circle"
- when drawing a 2-D figure (e.g., a circle) to represent a 3-D object (e.g., a ball), think of the region inside the circle as inside the ball rather than as the surface of the ball
- in trying to represent what an object is rather than how it happens to look, may draw what they know to be there; e.g., they show a hidden handle on a cup, or draw more sides of a cube than one could possibly see
- will often show a mixture of viewpoints in the same picture, e.g., side view of the legs of the table and top view of the table top and the items on it

These are the learning challenges for the Describing phase.

During the Describing Phase

Through their own physical and perceptual action on spatial configurations focused on interpreting, describing, and representing the parts making the whole, students make sense of the spatial relationships within figures, objects, and arrangements and in the visual representations of them.

As a result, they identify the features of particular figures *(This has four sides and two of its sides are equal.)* and objects *(This has six faces and they are all rectangles.)* and construct 3-D meanings for the 2-D representations of 3-D that are conventional within their culture.

Also as a result, students pay attention to the shape and placement of component parts when they draw, match, make, and copy things and are able to think of objects in positional relationship to each other rather than in relation to themselves.

By the end of the Describing phase, students typically:

- respond to a request to "tell me about the shape of this ..." using language such as "flat," "curved," "side," "round," "face," "edge," "square," "angle," "base"
- compare and contrast geometric figures
- are able to identify the faces, edges, and vertices of a geometric object and hence select component parts to make it in various forms (skeletal, hollow)
- understand that the word "shape" refers to or signifies both a 2-D and 3-D attribute and so understand, for example, that a cube and a square are different shapes and have different names even if they cannot recall the names
- when using a 2-D figure (e.g., a circle) to represent a 3-D object (e.g., a ball), interpret the region inside the circle as representing the surface of the ball
- match the 2-D figures with the faces of standard 3-D objects
- select nets that have the right component parts to match a simple object
- pay attention to the shape and placement of component parts when they interpret and make drawings
- observe the component geometric parts within pictures and patterns and the movement needed to produce them
- attempt to produce visual reality in drawings by only drawing the objects or parts of objects that can be seen
- rearrange and combine a few shape pieces (e.g., tangrams) to make another specified shape, such as a square
- repeat multiple copies of a figure in a systematic way to create a pattern
- recognize repetitions of the same shape embedded within arrangements and patterns
- identify component parts to show that a shape or arrangement is symmetrical
- are able to describe one thing being between others and put key features in order on a map
- attempt to show a bird's-eye view of familiar settings with a "rough" sense of proximity

What is the Diagnostic Map for Geometry and Space?

How students currently think about spatial ideas will influence how they respond to the activities provided for them, and hence what they are able to learn from them. As students' thinking about Geometry and Space develops, it goes through a series of characteristic phases that are described in this Diagnostic Map. Recognizing these common patterns of thinking helps to interpret students' responses to activities, to understand why they seem to be able to do some things and not others, and also why some students may be having difficulty in achieving certain outcomes while others may not. The Diagnostic Map also helps teachers to provide the challenges students need to move their thinking forward, to refine their half-formed ideas, to overcome any misconceptions they might have and hence to achieve the mathematical learning goals of Geometry and Space.

See over

Diagnostic Map: Geometry and Space cont.

Describing Phase

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These are the learning challenges for the Describing phase.

During the Describing Phase

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As a result, they identify the features of particular figures *(This has four sides and two of its sides are equal.)* and objects *(This has six faces and they are all rectangles.)* and construct 3-D meanings for the 2-D representations of 3-D that are conventional within their culture.

Also as a result, students pay attention to the shape and placement of component parts when they draw, match, make and copy things and are able to think of objects in positional relationship to each other rather than in relation to themselves.

By the end of the Describing phase, students typically:

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Analyzing Phase

Most students will enter the Analyzing phase between 8 and 10 years of age.

As students move from the Describing phase to the Analyzing phase, they:

- may think of a figure as a "picture" of a shape and so may, for example, think of a slant parallelogram as a rectangle looked at from "the side" (as in 2-D drawings of 3-D)
- may keep the same name for a transformed figure; e.g., a rhombus is just a pushed over square so it is still a square, an ellipse is a stretched out circle
- may use descriptive terms in ambiguous or incorrect ways; e.g., using "side" to mean "on the side" as distinct from "top" or "bottom"
- may still respond to figures by their overall appearance, and may therefore not recognize a shape when it is presented in an unfamiliar orientation; e.g., a square drawn "on its point" may not be recognized as a square, others will say that will be a square "if you turn it around"
- will be aware of some properties related to a common figure, but these properties may continue to play no detectable role in the recognition of the figure and students do not generally call upon properties to justify why a figure is or is not in a particular class
- may give vague descriptions that could apply to a number of different shapes, perhaps focusing on only one feature of a figure or object
- although now understanding which objects or parts of objects are seen or not seen, do not yet understand how they are seen (e.g., shape, orientation, size) and so have difficulty making their drawings "look right"
- even when provided with a drawing to copy, are influenced by how they think about or describe the object to themselves, so if told a drawing is a cube, they make the top face more square, if told it is a house they make the top more slanted
- select or draw nets that have the right component parts to make a particular object, but often ignore the relationship of the parts of each other so do not position them correctly to fold into a net
- may have little overall sense of relative position or scale in their plans and maps; e.g., they may draw their own desk larger than those of other students in the class

These are the learning challenges for the Analyzing phase.

During the Analyzing Phase

As students consciously compare and contrast spatial configurations, they form generalizations about relationships both within and between figures, objects, movements, and arrangements. Through their own experimentation, they realize that when an object or arrangement is transformed, relationships between its component parts may be preserved or not, and they try to visualize "what happens" when things are represented or moved.

As a result, students establish that shapes and movements they recognize as in the same class have features in common, thus the term "triangle" can now be interpreted as a collection of properties (a closed figure with three sides) that can be represented by many figures.

Also as a result, students try to ensure that desired relationships are preserved when they make (e.g., produce a net of an object, make a scaled copy), represent (e.g., draw a map or a diagram of an object), or move things (e.g., look from a different view, fold and unfold, turn).

By the end of the Analyzing phase, students typically:

- give a detailed list of properties in their descriptions of shapes, confidently asserting, for example, that rectangles always have four sides and always have right-angled corners
- select figures and objects based on geometric descriptions such as "have five faces and nine edges"
- know from the properties of a rectangle that a slant parallelogram cannot be a rectangle even though it is what a rectangle face on a block "looks like" from "the side"
- understand that lines drawn obliquely to the horizontal suggest depth and incorporate this into their drawings of objects
- use mathematical conventions to represent objects in different types of drawings
- match suitable nets to prisms and pyramids that are actually present (not drawn) by considering the shape and placement of the component parts
- produce their own nets for geometric shapes that they can see and touch
- visualize the folding process to say which of a number of potential nets that have the right number and shaped components will actually fold up to form a cube or prism
- select and build arrangements of geometric figures to match information in drawings and plans
- describe characteristic features of mirror symmetry; e.g., may explain that for mirror symmetry matching parts of figures are the same distance away from the mirror line
- visualize and reproduce the folds and cuts needed to produce symmetrical designs
- explain why they think a shape will not tile by focusing on the corners
- identify the particular rotations, reflections, and translations that relate to the component parts of simple arrangements and patterns
- understand that when figures are rotated, reflected, and translated, the position and/or orientation change but the size and shape do not, so the original figure can be superimposed on the transformed (or moved) figure
- understand that when figures are enlarged or reduced, the shape stays the same but the size changes (the position and/or orientation may change); that is, "scaled" figures keep the same shape and so look "the same but smaller" or "the same but bigger"
- understand the use of "scale" on a map to preserve proximity between things being represented; that is, to show the relative distance between things
- recognize and use a top view to represent familiar locations on plans using order and relative proximity among landmarks

Relating Phase

Most students will enter the Relating phase between 11 and 13 years of age.

As students move from the Analyzing phase to the Relating phase, they:

- may include irrelevant features as properties of families of shapes if they have only experienced shapes in upright orientations; e.g., they may think that the "top" and "bottom" sides of a trapezoid have to be parallel
- may have a correct verbal description for a concept and "know" they should use it but be unable to override a strong visual image or prototype; e.g., they may reject a square resting on its point as a square, even though it fits their definition
- may not generally understand that one class of shapes can be included in another, so that while they can be taught to recite that squares are special rhombuses, most do not understand why or how
- may have a well-developed concept of a particular shape, but their definitions may not provide sufficient features to define the shape; e.g., they may say that a rectangle is a shape with four sides and opposite sides the same length
- may define a class of geometric shapes and include many of the known features, not simply a sufficient set, thus providing redundant information
- may see properties as distinct from each other and so not see that some properties are a consequence of others; e.g., if a quadrilateral has four right angles it must have opposite sides the same length
- cannot typically coordinate all the components and measurements needed to plan a net completely from imagination or from specifications (except for simple well-rehearsed objects)

These are the learning challenges for the Relating phase.

During the Relating Phase

Students develop coordinated mental representations of spatial configurations in relation to their component parts enabling them to mentally manipulate and transform figures, objects, and arrangements. Through investigating *properties* of shapes and movements and inter-relationships between them, their use of visual images becomes constrained by their more abstract verbal knowledge of the properties.

As a result, students are able to visualize the result of systematically moving or folding figures or moving objects or themselves in relation to an object and to represent transformations. They also integrate distance and direction in their descriptions of paths and locations and can represent them on coordinate systems.

Also as a result, students come to recognize relationships between properties and between common classes of shapes; e.g., *This square is also a rectangle because it has all the properties of a rectangle*

By the end of the Relating phase, students typically:

- understand what a definition is and use counter examples to show that a definition such as "a rectangle is a shape with four sides and opposite sides the same length" is not adequate because it does not exclude some shapes that are not rectangles
- use properties to convince themselves and others why a figure or object belongs to a class; e.g., *This is a square because it has four equal sides even though it is not resting on a "flat bottom."*
- understand that knowing just a few properties of a figure or object enables us to work out (deduce) other properties
- understand relationships between properties of a figure; e.g., if a triangle has two equal angles then it has two equal sides
- understand class inclusion and so can classify figures and objects hierarchically; e.g., all squares are rectangles but not all rectangles are squares
- produce their own nets, considering in advance the level of precision needed to ensure the shape is correct in form and size, where tabs will be, and so on
- predict which face on nets will match which face on corresponding objects
- predict the effect of particular movements (translations, rotations, and reflections) on the orientation and position of figures and objects
- visualize an object or scene in different orientations, drawing other possible views of an object from information in 2-D drawings