



First Steps in Mathematics

Canadian Edition

A research-based diagnostic framework:

- * describing the developmental phases all students pass through as they learn mathematics
- * including validated diagnostic tasks that accurately identify student progress
- * outlining learning activities that target developmental triggers proven to advance student learning

Helping teachers see mathematics through the eyes of a learner.

PEARSON

Professional Development

From Research to the Classroom

In 1995, a team of university researchers and elementary classroom teachers were commissioned to develop a professional development resource that shows teachers how students learn mathematics. The development team began with an extensive review of international research, which represents thousands of student samples, cited in a 60-page bibliography. After augmenting this research with more than 10 000 of their own student samples, two key ideas became clear:

- 1) Students all over the world are experiencing the same mathematical misconceptions.
- 2) Big conceptual “phases” underpin the behaviours and understandings commonly held by all students.

In 2004, the development team published *First Steps in Mathematics* (FSiM) to describe these misconceptions and provide Diagnostic Tasks to expose them. FSiM outlines not only the conceptual phases but also key understandings and activities to advance students through them. Two thousand additional work samples were part of its validation process and, in 2005, its findings were replicated with 500 Canadian students. FSiM provides teachers with strategies proven around the world to advance student learning. It’s research that makes a difference.



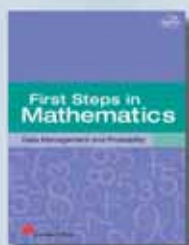
Number Sense



Measurement



Geometry & Shape



Data Management & Probability

First Steps in Mathematics Components

- ✓ **Professional Development**
 - 2-4 Day Coach Courses
 - 4-7 Day Facilitator Courses
 - 1-Day Administrator Course
- ✓ **Teacher Resource Books**
 - Diagnostic Map
 - Key Understandings
 - Sample Learning Activities
- ✓ **Course Books**
 - Diagnostic Tasks
 - Student Samples
 - Course Notes and Activities
- ✓ **Illustrated Poster of Diagnostic Map**

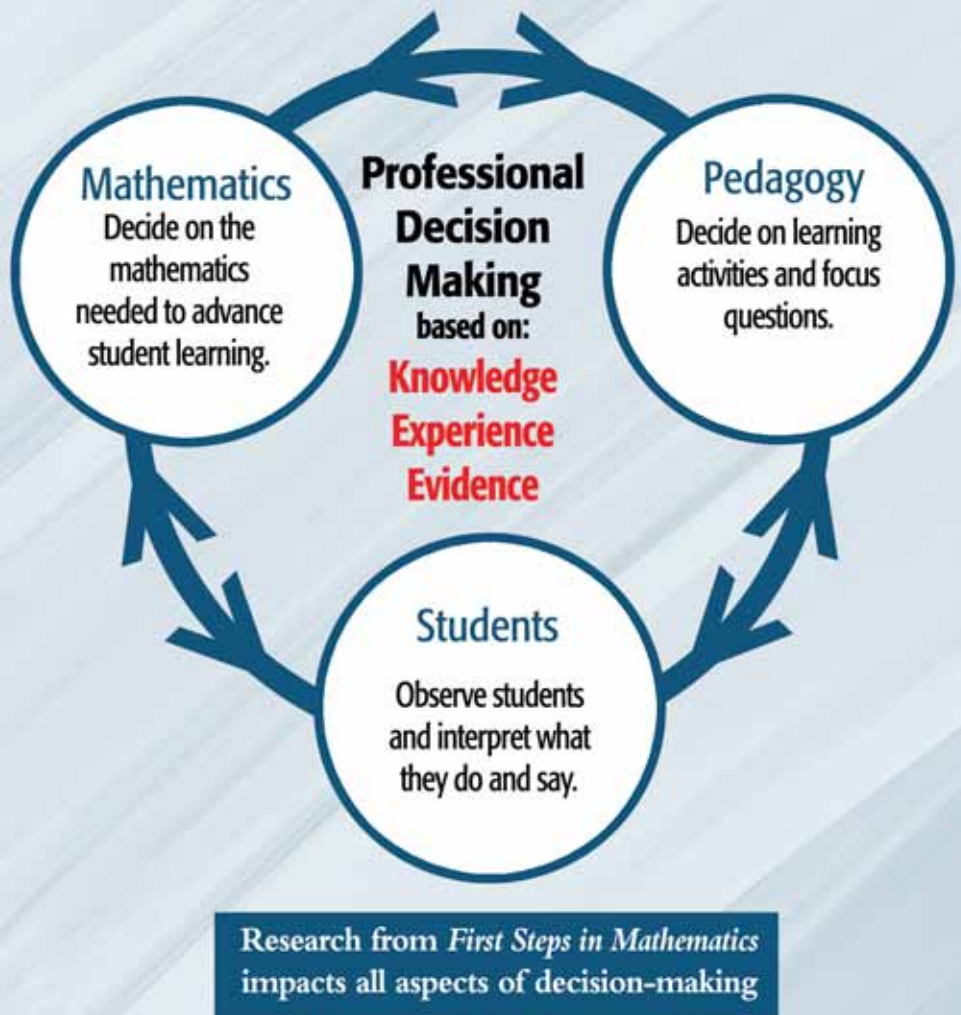
t that Makes a Difference

Empowering Teachers with the Power of Research

Every day, teachers make countless decisions that significantly impact student learning — decisions around *Mathematics, Pedagogy* and *Students*: What should I focus on when teaching mathematics? How should I teach it? What learning do students really need?

Research has the power to shape good decisions. The more *Knowledge* teachers have, the more *Experiences* they assimilate. The more sophisticated their methods for accumulating and evaluating *Evidence*, the greater the impact the teachers' decisions will have.

The research presented through *First Steps in Mathematics* enables teachers to make better decisions — informed decisions that ensure all students achieve deep understanding in mathematics. Decisions that will make a real difference.



First Steps in Mathematics helps teachers answer three main questions:

1. What is the path students travel as they learn mathematics?
2. How can teachers accurately diagnose student learning?
3. How can teachers advance student learning?

A Diagnostic Map that A

Question 1: What is the path students travel as they learn mathematics?

Diagnostic Map: Number

Emergent Phase
Most students will enter the Emergent phase between 3 and 5 years of age.

Matching Phase
Most students will enter the Matching phase between 5 and 8 years of age.

Quantifying Phase
Most students will enter the Quantifying phase between 8 and 11 years of age.

During the Emergent Phase
Students focus on small amounts of physical materials, learning to distinguish small collections by size and increasing precision and duration in them. They also learn to imagine and count the number associated with their collections and to distinguish between objects from other objects. There is a growing awareness of what is the same about the way numbers are used and students to describe collections and what is different between collections labeled with different numbers.

By the end of the Emergent phase, students typically:

- use fingers, "number" and the same to describe differences between small collections of the objects and between easily compared quantities.
- analyze whether an indicated change in a collection or quantity will make it larger, smaller or same in size.
- describe what number describes how many objects.
- describe numbers from other words or objects.
- use a 1 to 10 game box to count small collections and label correct number names to each collection.
- connect the different ways to represent collections of one, two and three with the number words "one, two, three."
- understand a request to share in a social setting and

While analyzing student work, the *First Steps in Mathematics* research team identified that students move through predictable phases when they learn mathematics. These developmental phases were then organized and described in a Diagnostic Map.

The Diagnostic Map details the developmental phases that help teachers understand:

- why students seem to be able to do some things and not others
- why some students may be experiencing difficulty while others are not
- why some students have been successful in the past but suddenly experience difficulty
- how a student's response fits within the larger learning continuum
- what learning opportunities students need to move their thinking forward, refine their preconceptions, and overcome any misconceptions

Diagnostic Map: Number

Partitioning Phase
Most students will enter the Partitioning phase between 6 and 9 years of age.

Factoring Phase
Most students will enter the Factoring phase between 9 and 11 years of age.

Operating Phase
Most students will enter the Operating phase between 11 and 13 years of age.

During the Partitioning Phase
Students focus on small amounts of physical materials, learning to distinguish small collections by size and increasing precision and duration in them. They also learn to imagine and count the number associated with their collections and to distinguish between objects from other objects. There is a growing awareness of what is the same about the way numbers are used and students to describe collections and what is different between collections labeled with different numbers.

By the end of the Partitioning phase, students typically:

- use fingers, "number" and the same to describe differences between small collections of the objects and between easily compared quantities.
- analyze whether an indicated change in a collection or quantity will make it larger, smaller or same in size.
- describe what number describes how many objects.
- describe numbers from other words or objects.
- use a 1 to 10 game box to count small collections and label correct number names to each collection.
- connect the different ways to represent collections of one, two and three with the number words "one, two, three."
- understand a request to share in a social setting and

The Diagnostic Map outlines the key indicators and the consequences of a student's understanding and growth. This information is crucial as teachers make decisions about what to teach, to whom, and when to teach it.

Assesses... and Instructs

Partitioning Phase

Most students will enter the Partitioning phase between 6 and 9 years of age.

Each developmental phase has three sections

Beginning

As students move from the Quantifying phase to the Partitioning phase, they:

- often cannot decompose into parts numbers that they cannot visualize or represent as quantities, so have difficulty in partitioning larger numbers to make calculation easier; for example, students need to count forwards or backwards by ones to find the difference between 25 and 38
- often use strategies based on materials, counting on or counting back to solve addition and subtraction problems, but do not link these strategies or different problem types to a single operation (either + or -)
- may be unable to use the inverse relationship between addition and subtraction to choose the more efficient of counting on or counting back for solving particular problems

The first section outlines the preconceptions, partial conceptions, or misconceptions that may exist as students move from the previous phase and enter the current one. These become the teaching emphases and learning challenges needed to progress through this phase.

During

During the Partitioning Phase

Students come to see the significance of whole numbers having their own meaning independent of particular countable objects. They learn to use part-whole reasoning without needing to see or visualize physical collections.

As a result, students see that numbers have magnitudes in relation to each other, can interpret any whole number as composed of two or more other numbers, and see the relationship between different types of addition and subtraction situations.

The second section describes the ideas students are coming to understand during the phase. It shows what students are thinking and doing, and why.

At the End

By the end of the Partitioning phase, students typically:

- can compare whole numbers using their knowledge of the patterns in the number sequence, and think of movements between numbers without actually or mentally representing the numbers as physical quantities
- make sense of why any whole number can be rewritten as the addition of other numbers
- partition at least two- and three-digit numbers into standard component parts ($326 = 300 + 20 + 6$) without reference to actual quantities
- count up and down in tens from starting numbers like 23 or 79
- write suitable number sentences for the range of addition and subtraction situations
- use the inverse relationship between addition and subtraction to make a direct calculation possible; for example, re-interpret $43 - 27$ as "what do you have to add to 27 to get 43" and so count on by tens and ones

The third section details the typical thinking and behaviours of students at the end of the phase. Together with the Beginning section of the next phase, these indicators help teachers identify the next steps needed to advance learning.

Targeted Teaching Strate

Question 2:

How can teachers accurately diagnose student learning?

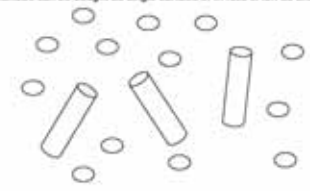
A series of short **Diagnostic Tasks** target key understandings and expose critical misconceptions that typically interrupt mathematical learning.

Validated through extensive research with students, these tasks, along with ongoing classroom observation, enable teachers to accurately locate students on the Diagnostic Map.

Candies

Name _____ Year/Grade _____ Date _____

Candies can be bought as single candies or in rolls of ten as shown here.



How many candies are shown here? _____

Name _____

Year/Grade _____ Date _____

Up To And Through The Hundreds

Write the numbers to the end of the boxes.
Begin at 91 and count by ones to the end of the boxes.

91	92	93									

Write the numbers to the end of the boxes.
Begin at 491 and count by ones to the end of the boxes.

491	492	493									

© Western Australian Minister for Education, 2005. Course book 137

CIRCLE THE BIGGEST

Name _____ Year/Grade _____ Date _____

1. Circle the biggest number:
37 **370**
How do you know it is bigger? _____

How many times bigger is 37? _____

2. Circle the smallest number:
647 **6470**
How do you know it is smaller? _____

Empty Boxes

Name _____ Year level _____ Date _____

What numbers and symbols would you use on the calculator to solve the following problems?

$17 + \square = 36$ _____

$\square - 27 = 34$ _____

$35 + \square = 16$ _____

$43 - \square = 18$ _____

$488 + \square = 842$ _____

$283 + 674 - \square$ _____

$\square - 15.75 = 12.43$ _____

© Western Australian Minister for Education, 2005. Course book 134

What Canadian participants are saying

about their *First Steps in Mathematics* Professional Development experience

“First Steps in Math training has changed my thinking about the way students learn mathematics. No longer do I identify students in a split grade mathematics class as grade 3 or grade 4 students. I now identify each student with the phase of development they are currently in and understand what mathematics is needed to teach and move them along through the next phase.”

First Steps in Math provides each student with the opportunity to address each misconception by challenging their current knowledge and understanding of mathematics, while providing meaningful mathematical experiences directly related to their individual needs.”

Tim Eirich
Curriculum Consultant
Prairie South School Division #210



First Steps in Mathematics

Professional Development Courses

For Kindergarten through Grade 8 Educators

Coach Course

- Provides in-depth coverage of the First Steps® in Mathematics instructional framework
- Builds capacity in coaches and teacher leaders, as they support teachers in effective instruction, and in the implementation of First Steps®
- Two or four days depending on strand
- Upon completion, coaches will be certified as users and providers of ongoing support in First Steps® and be able to model the use of the First Steps® resources within their district
- Coaches are entitled to order additional resource books at a 20% discounted price

Facilitator Course

- Develops teacher leaders to provide First Steps® professional development sessions to other teachers within their district.
- Introduces participants to the resource first as a learner, then as a facilitator
- Four or seven days depending on strand.
- Upon completion, facilitators will be certified as users, presenters, and providers of ongoing support in First Steps®.
- Facilitators are entitled to order additional resource books at a 20% discounted price

1-Day Administrator Course

This course, designed specifically for Administrators, outlines the foundational beliefs of *First Steps in Mathematics* and provides an overview of how the components work together for a strand. In addition, it offers practical strategies and activities Administrators can use to support and maximize the impact of the resource.



For more information, call 1-888-867-7772 or email:
pearson.learning@pearsoned.com

www.pearsonprofessionalllearning.ca

ISBN 13: 9780132046473
ISBN: 0132046474

S08-214-BC