Diagnostic Map: Probability—Measuring Chance

Number Sense Phase: *Matching*

Enter: 3–5 years Exit: 5–6 years

- Students use one-to one matching to judge quantity.

Measurement Phase: *Emergent*

Enter: 2–3 years Exit: 5–7 years

Students initially attend to the overall appearance of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small), rather than describing comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgments of size.

As a result, students begin to understand and use the everyday language of attributes and comparison, differentiating between attributes that are obviously perceptually different.

Number Sense Phase: *Quantifying*

Enter: 5–6 years Exit: 6–9 years

- Students trust the count to describe quantity without variance.

Measurement Phase: *Matching and Comparing*

Enter: 5–7 years Exit: 7–9 years

Students use one-to-one matching to directly compare things. They match in a conscious way to decide which is bigger by using familiar, readily perceived and distinguished attributes, such as length, mass, capacity, and time. They also repeat objects, amounts, and actions to decide how many fit (balance or match) a provided object or event. Until students understand the significance and invariance of the count, they cannot really understand the use of counting to measure size.

As a result, students learn to *use counting to directly compare* things so as to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as these: How long? How tall? How wide? How heavy? How much time? How much does it hold? And they know what to do when explicitly asked to measure something.

Number Sense Phase: *Partitioning*

Enter: 6–9 years Exit: 9–11 years

- Students use additive thinking to deal with many-to-one relations.

Measurement Phase: *Quantifying*

Enter: 7–9 years Exit: 9–11 years

Students connect the two ideas of directly comparing the size of things and of deciding "how many fit," and so come to understand that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared *without directly matching them*.

As a result, students trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

Number Sense Phase: *Factoring*

Enter: 9–11 years Exit: 11–13 years

- Students think both additively and multiplicatively about numerical quantities.

Measurement Phase: *Measuring*

Enter: 9–11 years Exit: 11–13 years

Students come to understand *the unit as an amount* (rather than as an object or as a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and *trust the measurement as a property or description* of the object being measured, something that does not change as a result of the choice or placement of units.

Number Sense Phase: *Operating*

Enter: 11–13 years Exit: —

- Students can think of multiplication and division in terms of operators, and reason proportionately.

Measurement Phase: *Relating*

Enter: 11–13 years Exit: —

Students come to trust measurement information, even when it is about things they cannot see or handle, and to understand measurement relationships, both those between attributes and those between units.

As a result, students work with measurement ***information*** and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.

Diagnostic Map: Probability

In Probability, students extend what they know about measuring *perceptual* attributes to measuring the *chance* something is likely to happen.

In this way, it follows the same development as Measurement: students become aware of the attribute (Could it happen? Won't it happen? Is it likely?); they compare and order from more to less of that attribute (Which is more likely? Equally likely?); they measure the attribute by comparison with a unit (being able to say how likely), and then to the standard unit (placing events on a scale from 0 to 1); they understand the relationships that enable us to indirectly work out the likelihood.

However, in another way, Probability does not follow the same development as Measurement. The fact that measuring chance is not a perceptual attribute makes it more abstract. As a result, when it comes to this new attribute, there is a lag when students move through the phases of development for Measurement.

Probability: Measuring Chance *Emergent*

Enter: 2–3 years Exit: 7–9 years

Students develop awareness that some things are more and less likely to happen and begin to use some of the comparative language of their communities to describe likelihood.

As a result, they use this type of language themselves and describe familiar, easily understood events as being more or less likely, e.g., Mom said we *might* go to grandma's after school; we are *more likely* to go home than to grandma's; we *usually* go home after school.

By the end of the Emergent phase in Probability, students typically

- are beginning to show that they recognize an element of chance in many things that are a part of their lives
- understand expressions such as "will happen," "won't happen," and "might happen"
- are able to distinguish impossible events from events that are possible but unlikely

But, as they enter the Matching and Comparing phase in Probability, they

- distinguish between certain and uncertain events, but may not realize that certainty must also include events that are certain not to occur
- may be unable to distinguish equally likely events, e.g., may assume all colours are equally likely to appear when given a four-colour spinner with unequal sectors
- may understand that some things are more likely than others, but not be able to provide relevant reasons why events might be more or less likely to occur (e.g., believe they will spin a 6 because 6 is their favourite number)

Probability: Measuring Chance *Matching and Comparing*

Enter: 7–9 years Exit: 9–11 years

Students draw on their experience to describe familiar things as more or less likely. They use expressions such as "very likely," "less likely," "equally likely," and "quite unlikely."

As a result, they are able to directly compare and order events from more to less likely and are able to justify their decision with relevant reasons.

By the end of the Matching and Comparing phase in Probability, students typically

- construct simple experiments and use counting to determine which event is more likely
- understand that that certainty includes those events that must happen and those that cannot happen
- understand what it means for simple events to be equally likely, e.g., can see why a spinner with four equal sectors is equally likely to stop on any colour, and that one divided into unequal sectors will not
- can list all possibilities for straightforward situations when prompted

But, as they enter the Quantifying phase in Probability, they

- may be uncritically influenced by other dominant features when ordering objects by likelihood, e.g., may be influenced by personal preference or personal experience and so say, "It is less likely to rain tomorrow because it never rains on my birthday" or "I'm more likely to roll a 6 because I always roll a 6"
- may construct an experiment to determine likelihood, but be casual about ensuring fairness, even to the point of altering or fixing outcomes to produce the predicted results

Probability: Measuring Chance *Quantifying*

Enter: 9–11 years Exit: 11–13 years

Students connect the idea of likeliness to the frequency of an event. They come to understand that repeated trials provide a count that can predict future likeliness. This count enables two things to be compared without directly comparing them.

As a result, they trust information gained from repeated trials as an indicator of probability and are prepared to use this to order events and determine how likely they are.

By the end of the Quantifying phase in Probability, students typically

- draw on personal experience to compare and order a variety of chance-related events and order them along a continuum that acts as an informal scale
- draw on numerical information alone to decide whether two simple events are or are not equally likely to occur
- are careful to ensure that an experiment is fair, e.g., that the sections of a spinner are equal or that the coin toss is consistent
- systematically list all possibilities, unprompted, to work out numerical probabilities for one-stage actions
- use experimental results to determine a range of possible outcomes and informally use relative frequency to estimate probabilities

But, as they enter the Measuring phase in Probability, they

- may be unable to create devices such as spinners or bags of coloured balls to produce specified orders of probability, e.g., make a spinner which is most likely to come up red and equally likely to come up blue or green
- may trust numerical information but may not be able to accurately order events where the total number of trials is not the same, e.g., might say that an event that occurs 8 out of 12 times is more likely to occur than an event that occurs 5 out of 7 because 8 is greater than 5

Probability: Measuring Chance *Measuring*

Enter: 11–13 years Exit: 15+ years

Students begin to quantify the chance of events occurring using probability as a measure for "how likely" or "how much more likely." They realize they can produce ratios by comparing the total number of occurrences to the total number of trials (experimental probability) or by comparing the number of desired outcomes to the number of possible of outcomes (theoretical possibility).

As a result, students understand that probability is the way we measure chance and that probability statements give a measure of how likely something is to happen.

By the end of the Measuring phase in Probability, students typically

- use a range of information sources to put things in order from least likely to most likely, e.g., use research data or experimental data to form conclusions
- understand that the greater the number of trials, the greater its reliability as an indicator of likelihood
- use their understanding of equivalent fractions to judge equally likely events
- interpret the 0 to 1 scale in general usage and understand why the probability that a toss of a fair die will produce 5 is one-sixth
- identify all the outcomes for two- or three-stage situations, e.g., rolling a die and tossing a coin

But, as they enter the Relating phase in Probability, they

- may not be able to overcome deep personal instincts about the likeliness an event should occur in spite of experimental or theoretical data
- may not recognize or trust calculations that would determine all possible outcomes for multiple-stage situations
- are able to simulate a situation where it would be difficult, costly, or inappropriate to generate real data, by designing simple experiments that replicate a significant aspect of the situation; they use their understanding of ratios and numerical probabilities

Diagnostic Map: Data Management

Number Sense Phase: *Matching*

Enter: 3–5 years Exit: 5–6 years

- Students use one-to one matching to judge quantity.

Measurement Phase: *Emergent*

Enter: 2–3 years Exit: 5–7 years

Students initially attend to the *overall appearance* of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small), rather than as describing comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgments of size.

As a result, students begin to understand and use the everyday language of attributes and comparison, differentiating between attributes that are obviously perceptually different.

Implications for Data Management

By the end of this phase, students are in a position to

- look at physical displays of familiar data and say which is most or has more
- sort and arrange data they have collected into familiar groupings
- count when asked to say how many in each group in a data display

But, students

- may count when asked to say how many in pre-arranged data, but focus on overall physical size rather than the numerical size of the group e.g., count 3 bananas and 5 strawberries, then say there are more bananas
- may focus on overall physical size rather than the numerical size of the group
- may say one group has more than another, but cannot say how much more
- may lay out objects, but lack the intention to compare

In Data Management, students use their understanding of Measurement to gather, compare, represent, and interpret data. In this way, students' understanding of Data Management is dependent upon their understanding of Measurement, which is dependent upon their Number sense.

As a result, the phases outlined in the Measurement Diagnostic Map should be considered when interpreting students' responses to Data Management activities. Doing so will help in understanding why some students may struggle to achieve certain outcomes while others do not.

The accompanying chart shows what students will be in a position to do and understand as they move through an appropriate program in Data Management, phase by phase.

Ultimately, Data Management becomes a quest to determine the likelihood or chance of something happening; it is intricately connected to Probability.

Number Sense Phase: *Quantifying*

Enter: 5–6 years Exit: 6–9 years

- Students trust the count to describe quantity without variance.

Measurement *Matching and Comparing*

Enter: 5–7 years Exit: 7–9 years

Students use one-to-one matching to directly compare things. They match in a conscious way to decide which is bigger by using familiar, readily perceived and distinguished attributes, such as length, mass, capacity, and time. They also repeat objects, amounts, and actions to decide how many fit (balance or match) a provided object or event. Until students understand the significance and invariance of the count, they cannot really understand the use of counting to measure size.

As a result, students learn to *use counting to directly compare* things: to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as these: How long? How tall? How wide? How heavy? How much time? How much does it hold? And they know what to do when explicitly asked to measure something.

Implications for Data Management

By the end of this phase, students are in a position to

- suggest counting as a way of answering data questions that focus on comparing collections, e.g., will suggest counting our pets to answer the question "Which pets are more popular?"
- use skip counting to say how many in a tally
- suggest direct comparison when prompted to record growth data, e.g., we can cut a streamer to match the sunflower plant each week to see how much it grows
- use counting to help construct their data display, e.g., construct a block graph by counting how many in each group, then counting how many squares to colour in
- understand the need for a baseline and space blocks regularly to allow comparisons to be made
- place direct measurement data in sensible sequences using a baseline, e.g., cut paper strips to fit around their heads and make a bar (column) graph by lining up the bottom of strips
- choose to count to compare the sizes of groups, without prompting
- look at a bar graph and say which bar has more based on its length

But, students

- may not attend to equal units when grid lines are not provided, e.g., they may create the correct number of pictures for each group, but not use the same size for each picture
- cannot construct a scale on the vertical axis to represent frequencies or measurements (although they can use a common baseline and label the horizontal axis with the groups)
- may not realize that the relative lengths of the bars relate to quantities in the collected data
- may not use a scale on the axis to tell how many, instead preferring to count

Number Sense Phase: *Partitioning*

Enter: 6–9 years Exit: 9–11 years

- Students use additive thinking to deal with many-to-one relations.

Measurement *Quantifying*

Enter: 7–9 years Exit: 9–11 years

Students connect the two ideas of directly comparing the sizes of things and of deciding "how many fit," and so come to understand that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared *without directly matching them*.

As a result, students trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

Implications for Data Management

By the end of this phase, students are in a position to

- see that, when organizing data, the categories can be reorganized without changing the overall total, e.g., to combine two brands of hamburgers, they need to add the two brands, or if they separate boys' and girls' responses for favourite take-out foods, the total must match the combined result
- see that they need to ensure that the data they collect is consistent
- produce and read pictographs or block graphs where each unit represents more than one piece of data
- produce simple two-way tables and Venn diagrams, partitioning totals between the cells or sections for straightforward data
- recognize that the length of bars in a bar graph can represent any numbers or measurements of data
- represent whole-number data in different ways, e.g., after measuring everyone's height, can produce a measurement graph that shows each child's height, or a frequency graph that shows the number of people of each height
- recognize that the length of bars and simple whole-number scales on the axis refer to quantities in the data collected, e.g., given a frequency graph about people's favourite take-out food, they know that the lengths of the bars represent the number of people naming each food in the survey
- use column and row headings to interpret what the numbers in simple two-way tables represent

But, students

- may be able to represent only whole numbers of units
- may not be able to work out how to represent the data when it is not a multiple of the unit
- may not realize that measurement data can be grouped
- may be unable to represent or read data using a continuous scale
- may not be able to convert to proportional measures to make comparisons
- may be unable to interpret the meaning between marked intervals on scales of frequencies or measures
- reading frequency data in two-way tables may not realize that to make sensible comparisons, the total frequencies need to be taken into account, e.g., students may say that more girls than boys like orange juice because 15 girls and 12 boys say they like it, but not realize that this information is misleading if there are 30 girls and 16 boys in the sample

Number Sense Phase: *Factoring*

Enter: 9–11 years Exit: 11–13 years

- Students think both additively and multiplicatively about numerical quantities.

Measurement *Measuring*

Enter: 9–11 years Exit: 11–13 years

Students come to understand *the unit as an amount* (rather than as an object or as a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and *trust the measurement as a property or description* of the object being measured, something that does not change as a result of the choice or placement of units.

Implications for Data Management

By the end of this phase, students are in a position to

- think carefully about the accuracy of their data and recognize that data collection is about measuring different aspects of a situation
- understand that they can group measurement data in their display
- create axes that show discrete or continuous quantities, including time scales
- use simple proportional comparisons when interpreting data in tables and graphs, e.g., half as many people prefer pizzas to hamburgers; the height of the wheat is three times higher than it was at the end of week 2

But, students

- may not recognize when they need to convert their data to fractions or percentages to make sensible comparisons

Number Sense Phase: *Operating*

Enter: 11–13 years Exit: —

- Students can think of multiplication and division in terms of operators, and reason proportionately.

Measurement *Relating*

Enter: 11–13 years

Students come to trust measurement information, even when it is about things they cannot see or handle, and to understand measurement relationships, both those between attributes and those between units.

As a result, students *work with measurement information* and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.

Implications for Data Management

By the end of this phase, students are in a position to

- choose from a wide range of measurement options when planning data investigations, using indirect measurements and creating measurement scales for non-standard attributes, e.g., they may create a scale from 1 to 5 to measure people's concern for environmental issues
- plan complex scales on axes to produce a wide range of graphs, including using class intervals, fractions, and percentages
- represent growth or change data over time by using a time scale, and approximate value within the intervals by joining the points when appropriate
- see that both axes can be made into number or measurement scales and used to show relationships between data, e.g., they could sketch a line graph to show how the amount of food on a plate varies over time at a buffet lunch as the plate is repeatedly filled with food, which is then eaten
- interpret displays showing relational information between measurements or frequencies
- interpret complex scales on graphs where not all scale markings are labelled